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⑥ Analytical Approximations

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⑦ by Cecil Hastings, Jr. and

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## Analytical Approximation

Offset Circle Probability Function: We consider the function

$$q(R, x) = \int_R^\infty e^{-\frac{1}{2}(p^2 + x^2)} I_0(px) p dp$$

in which  $I_0(z)$  is the usual Bessel function.

To better than .0017 over (0, 2),

$$q(2, x) \doteq .135 + .566 \left(\frac{x}{2}\right)^2 - .096 \left(\frac{x}{2}\right)^4 .$$

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### Analytical Approximation

Offset Circle Probability Function: We consider the function

$$q(R, x) = \int_R^{\infty} e^{-\frac{1}{2}(p^2 + x^2)} I_0(px) p \, dp$$

in which  $I_0(z)$  is the usual Bessel function.

To better than .0008 over (0, 3),

$$q(3, x) \doteq .011 + .231 \left(\frac{x}{3}\right)^2 + .654 \left(\frac{x}{3}\right)^4 - .329 \left(\frac{x}{3}\right)^6.$$

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## Analytical Approximation

Offset Circle Probability Function: We consider the function

$$q(R, x) = \int_R^\infty e^{-\frac{1}{2}(p^2 + x^2)} I_0(px) p \, dp$$

in which  $I_0(z)$  is the usual Bessel function.

To better than .0011 over  $(1, \infty)$ ,

$$q(R, R) \doteq \frac{R + .183}{2R - .388} .$$

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### Analytical Approximation

Offset Circle Probability Function: We consider the function

$$q(R, x) = \int_R^\infty e^{-\frac{1}{2}(p^2 + x^2)} I_0(px) p \, dp$$

in which  $I_0(z)$  is the usual Bessel function.

To better than .0004 over (0,1),

$$\begin{aligned} q(R, R) &= \frac{1}{2} \left[ 1 + e^{-R^2} I_0(R^2) \right] \\ &\approx 1 - .4921R^2 + .3212R^4 - .0966R^6 . \end{aligned}$$

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### Analytical Approximation

Offset Circle Probability Function: We consider the function

$$q(R, x) = \int_R^\infty e^{-\frac{1}{2}(\rho^2 + x^2)} I_0(\rho x) \rho d\rho$$

in which  $I_0(z)$  is the usual Bessel function.

To better than .0013 over (0,4),

$$q(4, 4-y) \doteq \frac{.551}{[1 + .187y + .055y^2 + .051y^3]^4}$$

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